Omissions and Measurement

Merlin Jetton

Introduction

Ayn Rand wrote: "A concept is a mental integration of two or more units possessing the same distinguishing characteristic(s), with their particular measurements omitted" (1990, 13). She did not include quantifiers. She did not write that *only* measurements are omitted in the main part of *Introduction to Objectivist Epistemology* (*ITOE*). However, she said that later. Per the expanded second edition, she said, "to establish the similarity by showing the characteristic is the same and *only* the measurements vary" (221; emphasis added).

I agree with her more general claim that concepts are grounded in similarities *and differences*, as elaborated by David Kelley in "A Theory of Abstraction." Kelley (1984) does an admirable job of elaborating Rand's framework. However, I contend that the framework is partly flawed in claiming that all differences between similar existents are ones of measurement.

Rand's claim was preceded by the example of *length*. "If a child considers a match, a pencil, and a stick, he observes that length is the attribute they have in common, but their specific lengths differ. The *difference is one of measurement.* In order to form the concept 'length,' the child's mind retains the attribute and omits its particular measurements" (1990, 11).

She gave only one other example—the concept *table*—before stating her claim. After giving a definition for it—a man-made object consisting of a flat, level surface and support(s), intended to support other, smaller objects—she says the definition retains and specifies the characteristic shape. The particular shape of the horizontal

The Journal of Ayn Rand Studies 7, no. 2 (Spring 2006): 383–405.

surface—round, square, or other—and its dimensions are omitted. Also omitted are the number and shape of the legs, the material of which the table is made and its utilitarian purpose.

Rand explains that the measurements that are omitted with regard to the material are those that differentiate one material from another. Similarly with regard to purpose, she explains that utilitarian requirements set limits on the dimensions of the table and rule out unsuitable materials, such as non-solids (12).

Were her justifications for material and purpose adequate? Is there some unit of measurement that differentiates between wood, plastic, steel, glass, and so forth, or combinations of different materials? Is there one that differentiates between the purposes of a dining table, an end table, a dressing table, a work table, and so on? In my view, material and purpose are only further differentiated by classifying, not measuring. Did Rand consider examples of a wide enough variety to justify her general claim of measurement omission? I think the answer to each of these questions is: No.

Many pages later in *ITOE*, Rand discusses concepts of consciousness. There she undercuts her previous claim of omitting only measurements:

> For instance, the concept "thought" is formed by retaining the distinguishing characteristics of the psychological action (a purposely directed process of cognition) and by omitting the particular contents as well as the degree of the intellectual effort's intensity. The concept "emotion" is formed by retaining the distinguishing characteristics of the psychological action (an automatic response proceeding from an evaluation of an existent) and by omitting the particular contents (the existents) as well as the degree of emotional intensity. (32)
>
> These concepts are formed by retaining their distinguishing characteristics and omitting their content. For instance, the concept "knowledge" is formed by retaining its distinguishing characteristics (a mental grasp of a fact(s) of reality,

> reached either by perceptual observation or by a process of reason based on perceptual observation) and omitting the particular fact(s) involved. (35)

Why is it omitting particular "contents" and "facts"? What happened to omitting only *measurements*? Moreover, it is not just concepts of consciousness for which something besides measurements is omitted—the same goes for many other concepts, especially higher level ones. More will be said on this later. It is time to consider the nature of measurement.

Measurement

Aristotle briefly talked about measurement as follows:

> We call a quantity that which is divisible into constituent parts of which each is by nature a one and a 'this'. A quantity is a *multitude* if it is numerable, a *magnitude* if it is measurable. (*Metaphysics* 5.13.1020a 7–10)

The terms "magnitude," "measure," and "measurement" are closely related. Sometimes they are used interchangeably. The glossary from Michell (1999) has the following:

> magnitude: a specific level of quantitative attribute (or quantity). For example, each specific length that any object might have is a magnitude of the attribute, length.
>
> measure: an estimate of the ratio of a magnitude of a quantity to a unit of the same quantity.
>
> measurement: the discovery or estimation of the ratio of a magnitude of a quantity to a unit of the same quantity.

Note that Michell presents "measurement" as the act rather than the result and "measure" as a noun rather than a verb. Still, measurement qua result is a ratio of one magnitude to another, the latter one used as a standard.

Probably the first rigorous attempt to illuminate the nature of magnitude was Book V of Euclid's *Elements*. In any case, his ideas have been very influential. One of his key ideas—profound and powerful—was that of ratio, which he defined as follows: "A ratio is a sort of relation in respect of size between two magnitudes of the same kind" (Euclid, *Elements* 5 Dfn. 3).[1]

Euclid's theory of ratios of magnitudes clearly influenced others over the centuries. Galileo's devotion to Euclid was intense. It strongly influenced Descartes as he created his revolutionary method of solving geometric problems by algebraic means. Newton explicitly defined number as the abstracted ratio of a quantity to a unit (Michell 1999, 32). Such influence continued with, among others, DeMorgan and later the eminent physicist James Clerk Maxwell, evidenced as follows:

> Quantities are abstract concepts possessing two main properties: they can be measured, that means that the ratio of two quantities of the same kind, a pure number, can be established by experiment; and they can enter into a mathematical scheme expressing their definitions or the laws of physics. A unit for a kind of quantity is a sample of that quantity chosen by convention to have the value 1. So that, as already stated by Clerk Maxwell,
>
> Physical quantity = pure number x unit.
>
> This means that the ratio of the quantitative abstract concept to the unit is a pure number. (14)
>
> Every expression of a quantity consists of two factors or components. One of these is the name of a certain known quantity of the same kind as the quantity expressed, which is taken as the standard of reference. The other component is the number of times the standard is to be taken in order to

make up the required quantity. (33)

Algebraically then, Maxwell's formula is M = R x U, where M is magnitude, R is a real number and U is the unit. For many, but not all, kinds of units R is positive. R could be negative for rate of change in velocity, for example.

Divide both sides of the equation by U. The result is: M/U = R. This formula highlights the significance of *ratio*. It shows the intimate connection between ratios and real numbers. The left side is the ratio; it equates to a real number. Consider an example using inches:

15 in. = 15 x (1 in.)

If we divide both sides of this equation by 1 inch, then simplify the ratio 15/1 to 15, we get the identity 15 = 15, since inches in the numerator and denominator of the ratio cancel out. We would get a similar result no matter what unit of measurement is used.

Magnitudes so constructed are subject to mathematical operations, addition being the main one. For example, if we took a weight of 10 pounds and added 5 pounds to it, we would get 15 pounds. If we took a rope 30 feet long and cut off 5 feet, we would get a rope 25 feet long. If we took a gallon of milk and evenly split it four ways, we would get four quarts.

Such mathematical operations rely on equal differences between successive integer values of the magnitude. "30 feet minus 5 feet equals 25 feet" relies on each foot of the 30, of the 5, and of the 25 to be identical. For ratio measures like length, ratios of values are also meaningful. For example, "30 feet divided by 5 feet equals 6" relies on said equal differences and using the same unit of magnitude in both the numerator and denominator of the ratio.

Magnitudes per one standard unit (of absolute scale) are easily transformed into magnitudes per a different standard unit simply by multiplying the former by a real number. For example, magnitudes in meters can be transformed to feet by multiplying by 3.2808399. The ratio of one magnitude to another is invariant of the unit used. For example, the ratio of the height of a rectangle to its width is the same, whether the magnitudes are in centimeters or inches.

Usually measuring is done with some kind of *gauge*. Lengths are measured with a tape measure or ruler, weights with a scale or balance, time with a clock or watch, liquid volume with a marked container, electric current with an ammeter. The gauge is an *external* device. This makes determining magnitude largely a matter of observation. The user simply reads the result. As long as the user is competent with the gauge, and recognizes the natural limits of its precision, the reading is exempt from subjective influence. Any other competent user in identical or similar circumstances will come up with the same reading. These conditions are not met when it comes to subjective sensations.[2]

Ordinal Ranking

According to Michell, prior to the mid-twentieth century, measurement in psychology was understood in the classical manner. For example, J. M. Baldwin in his 1902 *Dictionary of Philosophy and Psychology* wrote about *measurement* as follows: "In order that a concept be measured as a mathematical quantity without ambiguity, it is essential that the special attribute by which we measure it be conceivable as made up of discrete parts, admitting of unambiguous comparison *inter se*, with respect to their quality or difference in magnitude" (185). Similarly H. C. Warren in his 1934 *Dictionary of Psychology* defined *measurement* as: "The comparison of a quantitative datum of any sort with a fixed, enduring datum or standard of the same sort" (185).

Then the situation in psychology changed with S. S. Stevens leading the way. Over a period of about 30 years, he said and wrote repeatedly: "Measurement is the assignment of numerals to objects or events according to a rule." Since the 1950s, his definition of measurement has been the model for psychologists (15).

Psychophysicists like Weber (from 1830 on) and Fechner (from 1860 on) made popular "quantitative psychology" with the development of algebraic formulas for linking measurements of external stimuli to psychological intensities or sensations. The latter—relying on introspection—could be ranked with pretty small noticeable differences. Fechner took this as good enough to construe it as "measurement." Philosophically, he conceived the mind and brain as one. Predictably, there was a negative reaction from mind-body dualists. In addition, there were other strong objections: that the smallest noticeable difference is not constant on a linear or log scale and varies by person; and the subject's sensation cannot be observed. He responded vigorously, as did other psychophysicists later defending their own forms of quantifying. The debate went on for decades, with statistical theory entering the scene in the later years. Eventually the psychologists won the debate as far as they were concerned, in large part by ignoring the objections (79–108).

Stevens, famous for his psychophysics of sound, continued the push for an alternative concept of measurement. He was influenced by and helped popularize operational definitions, by which any concept is nothing more than a set of operations and is synonymous with those operations. For example, if a subject, based only on his hearing, judges that one sound is twice another, that ratio 2 is as valid as any other ratio 2 (162–90). Note, however, that such a ratio is not the result of dividing one number by another.

Stevens first called his psychophysical results "absolute judgment," then "numerical estimation," and finally "magnitude estimation." Most of this evolution was in the 1950s. His description of the method was that the subject brings the numbers with him, so to speak, and the experimenter needs only to provide the target stimuli to which the numbers are to be matched. Typical instructions to subjects were:

> You will be presented with a series of stimuli in irregular order. Your task is to tell how they seem by assigning numbers to them. Call the first stimulus any number that seems appropriate. Then assign successive numbers in such a way that they reflect your subjective impression. (Stevens 1971, 428)

Stevens even says, "Experience has shown that it is usually better *not to designate a standard*" (428; emphasis mine). Thus, Stevens was well aware of the differences between psychophysical experiments and measurement in the harder sciences. But he had another aim—to legitimize psychology as a science. Much of the prestige of the harder sciences comes from their being quantitative. Psychologists yearned for more prestige for their field and imitating the more quantitative sciences seemed to be a way. They wanted to regard psychological intensities or sensations as measurable. This was despite the existence of long-standing compelling arguments that they are not measurable in the classic manner. There are no exact numerical relationships between them and external stimuli, and psychological intensities have no precise quantitative units. The external stimuli may be measurable, but that does not imply that psychological intensities or sensations are. Our quantifying them is of "less," "equal," or "more," but not of "how many times more or less" with precision and consistency. For example, physical pain is not measurable in the classical manner. Plausibly it is because it has so many dimensions: intensity, duration, quality (e.g. stabbing versus chronic), how localized it is and where, and the degree to which the person tolerates it and attends to it.

I have no objection to psychophysics per se. However, trying to redefine measurement is objectionable, and Stevens's definition opened the door far too wide. Suppose you agree to taste ten flavors of ice cream and then rank them with the numbers 1-10, 10 being for the flavor you like best and 1 being for the flavor you like least. This procedure fits Stevens's definition. Does that mean that you like flavor #10 twice as much as flavor #5 or ten times as much as flavor #1? Does that mean you like flavor #10 more than flavor #9 to the exact same extent that you like flavor #7 more than flavor #6? The answers to these questions are "no." Equal differences between successive integer values of the magnitude do not hold for ordinal ranking.

For the measurement length, adding 2 inches to 4 inches makes 6 inches. There is no analogous "flavor unit" with each flavor being so many flavor units. There is no analogous operation by which one could convert one flavor to another and then another, e.g., chocolate to strawberry to vanilla, simply by adding or subtracting uniform "flavor units." In other words, chocolate, strawberry and vanilla are not commensurable. They are only congruous, or substitutes.

One could alternatively rank the flavors in reverse, with #1 as best and #10 as worst, but again the numbers assigned are not amenable to mathematical operations. If someone else agreed to taste the ten flavors and rank them, would such person rank each flavor with the same number you did? If you were to rank the same ten flavors in a few years, would you rank them in the same order? Except in rare circumstances, the answer to each of these questions is clearly "no." Such subjective influences do not occur with true measurement.

Note that the ranking could be done equally well with letters of the alphabet. All that is needed is a rule like "assign A to the flavor you like best and J to the flavor you like least" or vice versa. Like letters, assigned numbers are no more than labels. Letters do just as well in expressing the fact that you like one flavor more or less than another. It is also clear that there is no standard quantitative unit of measurement here. There is nothing comparable to an inch, an hour, a liter, feet per second, and so on. If there were such a unit, a good name for it would be a *yummy*.

To contrast the difference between true measurement and ranking using numbers, Michell (1999, 17) uses the terms "discover" and "assign."

> Measurement is the attempt to discover real numerical relations (ratios) between things (magnitudes of attributes), and not the attempt to construct conventional numerical relations where they do not otherwise exist. The difference would be most dramatically seen if the attributes were not actually quantitative. Then there would be no ratios to discover, and measurement would be logically impossible. Nonetheless, numerical assignments according to some rule could always be made to the objects or events involved. This highlights the logical distinction: the making of numerical assignments may be many things (e.g., useful, convenient, rewarding), but true (or false) is not one of them. On the

> other hand, to claim that my room is four metres long is to assert something which is either true or false.

This is an excellent distinction. The ice cream example is a case of trying to construct conventional numerical relations that do not otherwise exist.

True measurements are easily ranked, or ordered, because they have a basis in real numbers. Indeed, such measurements would be "well-ordered" in the mathematical sense. However, this does not work in reverse, since not all rankings are true measurements.

One commonly finds in psychology literature these days the following kinds of measurement or scaling:[3]

1. nominal: numbers on sports uniforms; social security numbers
2. ordinal: rank in graduation class or preference
3. interval: temperature; IQ
4. ratio: wealth; weight; distance

The first is mere labeling. For the second, differences are *relatively* meaningful—the differences are not based on a uniform unit—but the exact magnitudes are not. For the third, differences are *absolutely* meaningful—the differences are based on a uniform unit—but the exact magnitudes are not. Temperature in terms of Fahrenheit or Celsius is not a ratio measure since, for example, dividing a positive temperature by a negative one does not yield a meaningful result. The fourth, of course, meets the criteria for classical measurement. *Teleological* measurement—as Rand used the term—fits in the second category with "ordinal."

It should be clear by now that the first two fall far short of the criteria for the last two. To pretend or suggest that they are equivalent to true measurement is unwarranted. There is no unit of measurement. In short, "ordinal measurement" is an oxymoron.

Rand Revisited

Ayn Rand showed a good understanding of measurement at times. For example: "Measurement is the identification of a relationship—a quantitative relationship established by means of a standard that serves as a unit" (Rand 1990, 7). However, she did not use the concept consistently, often lapsing into *teleological measurement* and *ordinal measurement*. These are constructs. In other words, she sometimes used *measurement* in the manner of Stevens's definition, not the classical one.

Rand also used the term *unit* in two ways:

1. Member of a set or class: "A concept is a mental integration of two or more units which are isolated according to a specific characteristic(s) and united by a specific definition" (10).

2. The basis of measurement: "Observe that measurement consists of relating an easily perceivable unit to larger or smaller quantities, then to infinitely larger or infinitely smaller quantities, which are not directly perceivable to man" (8).

The second could be construed as a special case of the first, but they are not equivalent. In some places, her usage could be taken either way: "The ability to regard entities as units is man's distinctive method of cognition" (6). "Units are things viewed by a consciousness in certain existing relationships" (7). Whether or not these two meanings contributed to any confounding and ambiguity regarding "measurements omitted" is hard to say.

Regardless, a claim that it is *only* measurements that are omitted in concept formation implies that *every* kind of attribute is measurable. I think this has been shown to be false. Every kind of attribute would need a standard quantitative unit. If someone claims an attribute is measurable, the burden of proof is on him or her to say what this standard quantitative unit is.

The case is supported with the insightful, practical argument Norman Campbell gave in his 1921 essay "Measurement," paraphrased as follows. Some attributes and not all can be thus represented by numbers. When buying a sack of potatoes, I may ask what it weighs and what it costs. To those questions I can expect a number in answer; it weighs 20 lbs. and costs 89 cents. But I may also ask of what variety the potatoes are, and whether they are good cookers; to those questions I shall not expect a number in answer. The dealer may possibly call the variety "No. 11" in somebody's catalogue; but even if he does I shall feel that such use of number is not a real measurement, and is not of the same kind as the use in connection with weight or cost. What is the difference? Why are some attributes measurable and others not? It is this. Suppose I have two sacks of potatoes identical in weight, cost, variety, and cooking qualities; and I pour the two sacks into one so that there is only one sack of potatoes. This sack will differ from the two original sacks in weight and cost (the measurable attributes), but it will not differ from them in variety and cooking quality since these attributes are not measurable (Campbell 1988, 1771–72).

It was noted above that Rand weakened the case for "omitting measurements" in connection with concepts of consciousness. Let us test a wider variety of concepts, even ones that are not concepts of consciousness, to see how well the claim of "measurements omitted" holds up more generally. In other words, we will try to follow Rand's dictum of reducing a concept to its basis in particular facts (Rand 1990, 51).

Consider the concept *book*. Some attributes of books are measurable, e.g., height, width, and thickness. However, some important ones are not. Consider the language in which it, or part of it, is written—English, Spanish, German, C++ (the computer programming language), musical notation, mathematical notation. The last three are not ordinary languages. Yet all are congruous—all are ways of conveying ideas in writing. However, these languages are not commensurable. The differences between them that need to be omitted to form the concept *book* are qualitative, not measurable.

Consider also the content of the book. Some books are fiction and others are nonfiction. Types of fiction are mystery, romance, children's stories, etc. Types of nonfiction are history, science, mathematics, music, food recipes, etc. These various contents are congruous but not commensurable. The differences between them that need to be omitted to form the concept *book* are qualitative, not measurable.

Consider ice cream again. Suppose a young child has eaten ice cream multiple times and has developed the concept *ice cream*. She is quite capable of identifying something as ice cream or not. If she has not yet experienced frozen yogurt or sorbet, she may mistake them for ice cream. Such a mistake is excusable and should not negate the judgment that she has a concept of ice cream. Has the child "omitted measurements"? I think not. Further suppose she is still learning to count and has no idea what measurement is. She has attained her concept of ice cream by identifying its characteristic look, that it is cold, creamy and sweet, and its characteristic taste. The tastes differ among instances because of the particular flavor or the way it was made, but there remains a characteristic ice cream taste on account of the main ingredients. There are many differences among the various instances. Besides different flavors, some may include nuts, others not. Some are marbled, others not. But the child has treated each instance as a unit and integrated them into the concept ice cream. The most noticeable difference is flavor, which is a *qualitative* one. Again we would have to say most of these differences are *qualitative*. They are not of numerical attributes, hence not measurable, and it is logically impossible to have omitted measurements.

A defense of Rand's claim is that the child has *implicitly* omitted measurements. On the other hand, some claim this part of her theory is vague because she used "implicitly" so often with little explanation of the word.[4] I agree, and in my view this strategy does not bear scrutiny. It is fine to use "implicit" to mean a person does X, but the person only later becomes self-aware that he or she was doing X. It is fine to use "implicit measurement" to mean estimating length "by eyeball" as a substitute for using a tape measure. It is quite another matter to say that a person "implicitly" does X when the person lacks the ability to do X.

Her use of *omitted* is also vague. Does it mean the differences are recognized but disregarded or omitted because they are not even recognized?

Studies of children provide plenty of evidence against the claim that a child who can recognize that a pencil is longer than a match and a stick is longer than a pencil is capable of measuring, even implicitly.

> The formation of concepts according to Rand's formula of measurement omission requires a dimensional understanding of isolated attributes. The dimensions must be accessible but

> might be scalable only ordinally. A child knows size as a dimension when he regards big and little as attributes of a single kind; part identities have then become organized as a dimensional kind. He knows size as an ordinal dimension when he regards bigger and littler as necessarily opposing direction of difference. Preschoolers have only a fragmented understanding of these quantitative relations.[5] (Boydstun 1990, 36)

Fragmented indeed. Intermediate between being capable of knowing that one object is longer or shorter than another object and a grasp of measurement is *seriation*. This is Jean Piaget's term for the ordering or ranking of a few or several objects along a single dimension. His experiments with children, using such activities as sorting several rods according to length, show that most children are not competent at the task until age 6-8 years. All the 4-year olds failed. Some had partial success, getting 2-4 rods in the correct order. Most 5- and 6-year olds succeeded using trial and error. Only the 7- and 8-year olds were really competent—to take one rod at a time and put it into the correct place in the existing order without error (Gruber and Vonèche 1995, 385–86). Going beyond seriation to measurement requires using—not merely assigning—numbers to denote specific lengths based on an invariant standard quantitative unit.

Piaget and his colleagues tested children for their understanding of measurement in many ways. "For a long time children are content with visual comparison, and only later do they think of putting objects alongside each other to check their estimate, while the concept of a measuring rule which can provide a common measure arises still later" (Piaget, Inhelder, and Szeminska 1960, 28). Children aged 4-5 years are usually in the first stage the authors describe, and the last stage is usually not reached until 7-8 years old. While the stages are described in much greater detail, with sub-stages, the following is a summary of the results.

1. Reliance on perception. No notion of common measure. Disregard of common measure even when instructed.

2. Common measure recognized. Partial success in measuring

and partial reliance on perceptual judgment.

3. Full competency in measuring.

Some experiments tested a child's understanding of length or height or distance, often in problems designed so that measuring was the only reliable way of deciding that one line or object is longer or taller than or equal to another. Younger children were rarely capable of measuring even when given hints or training on how to do it (27–66). Other experiments were a little more complicated. For example, the child was shown two lines with one or both being bent. Again the younger children showed no or little measurement ability, even when given hints or training about how to use a short strip of paper to "step off" a length. By age 7-8, most children were capable of measuring (Piaget 1960, 104–27). [6]

Such experiments show a huge difference between perception of quantity and the *concept* of measurement, even the practice of measurement. In conclusion, it seems to me that to claim a child when first capable of recognizing that a pencil is longer than a match, and a stick is longer than a pencil, is "implicitly measuring" is quite a leap. I would say the child has grasped "unit" in the first sense given above, but not the second.

Gelman and Williams (1998) offer reasons why children find learning fractions (ratios)—required for measuring as said above—difficult. Fractions are not part of the knowledge structure of the counting numbers. The children put new learning at risk because they rely so much on what they already know and how that knowledge is organized in their minds. All of us interpret new experiences using what we know, not what we *will* know. Like Rand (1990, 42–43) often said, knowledge is contextual. Children are inclined to interpret input about fractions in light of what they think about counting—"numbers are what you get when you count" (Gelman and Williams 1998, 617). Gelman and Williams even claim that the idea of infinity is easier to learn than fractions. Infinity is consistent with the mathematical structure of counting—each number has a successor—whereas fractions are not (617–18).

From an Objectivist perspective, when the child learns about measuring, she must integrate what she knew about counting with her new experience. She must do that to properly understand measuring and how it relates to counting. Rand's examples involved only linear measurements. To understand 2- or 3-dimensional measurement, one must also know some arithmetic. Two examples are speed or spatial volume.

Suppose a child knows that a dog or rabbit can run faster than her. Does that mean the child implicitly knows how to measure speed? Unlike length, speed explicitly involves two magnitudes, distance and time. Thus to implicitly measure speed, the child would need to implicitly measure both distance and time, then implicitly divide distance by time.

Next we examine Rand's own justification for "teleological measurement" being measurement.

> Measurement is the identification of a relationship—a quantitative relationship established by means of a standard that serves as a unit. Teleological measurement deals, not with cardinal, but with *ordinal* numbers—and the standard serves to establish a graded relationship of means to ends.
>
> For instance, a moral code is a system of teleological measurement which grades the choices and actions open to man, according to the degree to which they achieve or frustrate the code's standard of value. The standard is the end, to which man's actions are the means. (Rand 1990, 33)

In her examples that precede the above ("thought" and "emotion," quoted above), she appeals to measurement on the basis of psychological intensity. In other words, she does what psychologists have done. She also uses "standard" here in two different ways. A standard for measurement is an invariant magnitude of the same kind that is being measured. Her "standard" for "teleological measurement" is of a very different sort. The implied sameness fails.

Ranking per se lacks the degree of precision that measurement has. In my own definition of *number*, I have used *definite* (meaning precise) to differentiate number from other kinds of quantification.

Imprecise ways of quantifying—some, several, greater than, handful, and so forth—are quantifying but without numbers (Jetton 1990, 2). By analogy, number is to measurement as ordinal ranking is to indefinite quantifying.

Rand again showed how far she stretched ordinary meanings when describing her theory of concepts as "mathematical." That is not wrong per se. Indeed, it is a normal part of learning and creativity. A great example of such stretching is her claim that love is measurable. She staked this on intensity—using liking, affection, and romantic love.

Accordingly, it is ranking, not measuring, as explained above. Moreover, instances of love are differentiated by more than just intensity. They are also differentiated by other qualitative factors—like why a person, object, or activity is loved.

I do not know whether she was aware of the debate going on in psychology or did this on her own. In any case the parallel exists. She also admitted to herself that her ideas about measurement and its omission were very tentative per the *Journals of Ayn Rand.* This entry was published after her death, and she probably did not intend it for publication (Rand [1997] 1999, 700–2). In contrast, nothing is said in *ITOE* about the tentative nature of her ideas.

Peikoff's *OPAR*

In *Objectivism: The Philosophy of Ayn Rand* (*OPAR*), Leonard Peikoff basically repeats the arguments that Rand made about measurement omission in *ITOE*. He reuses her examples of length and table. He raves about her discovery of measurement omission, calling it "momentous" ([1991] 1993, 82) and a "seminal observation" (83). He considers no wider variety of examples than Rand did. He comments:

> In her treatise, Miss Rand covers all the main kinds of concepts, relationships, and materials. In each case, she explains how the principle of measurement omission applies. (88)

> The process of measurement omission is performed by us by the nature of our mental faculty, whether anyone identifies it or not. To form a concept, one does not have to know that a form of measurement is involved; one does not have to measure existents or even know how to measure them. On the conscious level, one need merely observe similarities. (85)

> The answer to the "problem of universals" lies in Ayn Rand's discovery of the relationship between universals and mathematics. Specifically, the answer lies in the brilliant comparison she draws between concept formation and algebra. (89)

Peikoff's idea of measurement does not conform to its scientific sense either. Given what Rand said was omitted for concepts of consciousness, the first quote from Peikoff is an exaggeration. The second quote is non-objective—making an empirical claim with no empirical evidence. The paragraph is a non sequitur. If he could present some neurological or psychological evidence that this is a plausible hypothesis, then it would be time to listen. Meanwhile, it merely begs the question.

Consider the third quote from Peikoff. Why does he say that Rand's solution rests on her comparison with *algebra*? Both Peikoff and Rand stressed *measurement omission* far more heavily. Nevertheless, here I think that Peikoff and Rand got it right. She compared concept formation to algebra as follows:

> The relationship of concepts to their constituent particulars is the same as the relationship of algebraic symbols to numbers. In the equation 2a = a + a, any number may be substituted for the symbol "a" without affecting the truth of the equation. For instance 2 x 5 = 5 + 5, or: 2 x 5,000,000 = 5,000,000 + 5,000,000. In the same manner, by the same psycho-epistemological method, a concept is used as an algebraic symbol that stands for any of the arithmetical sequence of units it assumes. (Rand 1990, 18)

This is a great analogy, but she takes it too far in one respect. How are the various languages and contents of books considered above to be considered an "arithmetical sequence"? As she claims elsewhere, the symbol "a" and a concept are both open-ended. However, that does not imply the units of a concept form an arithmetical sequence.

The most striking aspect of Rand's analogy to me is the following parallel between (a) the letter "a" and numbers and (b) a concept and its referents. Both relationships are between quite different kinds, at least when the references are external existents. In the first case, a letter represents something quite different in kind—numbers. In the second case, an idea represents something quite different in kind—external existents.

Another Perspective

There is another, more complex, way of looking at concepts algebraically. It seems to have potential for a richer explanation of concept formation and use. It needs more thought and may warrant another paper, but the following is a start. A general algebraic expression of an n-dimensional function is:

$$y = f(x1, x2, x3, \ldots, xn)$$

For example, a function for the area of a rectangle would be area = f (length, width). More specifically it would be area = length x width. Area and speed, mentioned earlier, are 2-dimensional on the right side, so $n = 2$. With $n = 3$ and a rectangular packing box, volume = f (length, width, height). While most people hesitate when $n > 3$, it is an easy step mathematically.

This same general expression can be extended to concepts, though the ability to use numbers often declines, for example:

bird = f (animal, two legs, wings, feathers/down, beak, fly)

This is more abstract since $n = 6$. More dimensions could be added to increase n. For a particular bird, all these variables have particular values, some of them quantitative and others only qualitative. In cognitive psychology, the latter may be called *features* to note the difference (Kelley 1984, 7). Numerically the "y" for bird may be restricted to 1 or 0, meaning "bird" or "not a bird," depending on the values on the right side. There could also be logically preceding functions for variables on the right for some concepts. But let us return to the main topic.

Alternative Proposals

If the reader is convinced by the above arguments, what could replace "measurements omitted" in Rand's description or definition of a concept? Some candidates are:

1. measurements or non-measurable qualities omitted
2. differences of content or measurement omitted
3. details omitted
4. differences of detail omitted
5. leaving out those particulars wherein they differ

I like the fourth best for several reasons:

1. It includes "differences," which contrasts it with "similar" in the first part of her description or definition of "concept."

2. "Details" can cover both qualities and quantities.

3. It points to the fact that what we disregard in forming a concept are specific attributes (or further specifics about such attributes) of the units integrated by the concept that are not considered as fundamental. "Just give me the basics, not all the details" is a common request. "Details omitted" fits that request.

4. Rand amply described how regarding things as units and forming concepts gave us cognitive economy. "Details omitted" further appeals to the cognitive economy that concepts provide us.

I like the fifth a lot and took it from John Locke ([1690] 1952, III.iii.9).

This is not to deny that measurements are omitted with respect to some concepts—ones that have attributes that are quantitative to begin with, such as *length* and *triangle*. It simply recognizes that so many attributes are only (known to be) qualitative.

Rand's idea of similarity must also be challenged. She defined similarity in terms of sharing common dimensions, but differing in measure or degree (Rand 1990, 13). However, differences may be of type, not only measure or degree. In other words, they may be qualitative, not merely quantitative. For example, ice cream flavors differ in quality, not quantity. Airplane engines may be classified as propeller, jet, turbojet, or rocket. These types are similar in that they are means of propulsion for an airplane, but the difference is not a matter of measure or degree.

I even propose that quality is logically prior to quantity. A measurement is *of something*, that something being some quality.

Notes

1. Euclid conceived ratios as one whole number to another. The decimal system of notation we use today did not come into existence until more than 1700 years later. Book V does not even use numbers for magnitudes. It uses one line segment relative to another, using letters like A and AB to designate line segments.

2. There are some things I would consider measurements that do not use a gauge. Examples are average number of words per page in a book or population density (people per square mile). No physical gauge is used. However, they do not rely on *assigning* numbers, and the unit is clearly specified.

3. Probability and statistics are often used in psychology, too. It is not exactly classical measurement, but it is very close when the numbers come from counting or measuring—not mere assignment—and computing ratios.

4. Bryan Register (2000, 224–25) held that Rand was quite vague about her use of "implicit." Robert Campbell (2000, 214–15) endorsed Register's view and made further comments on the vagueness of her usage.

5. Boydstun (2004, 278) seems to approve of Rand's attempt to consider any ordinal ranking as a form of measurement. Of course, I do not. Boydstun writes:

> Like the common dimension for strength, it is a dimension consisting of nothing more than various substitution dimensions. The measurable dimension of *property* of a solid will be the hardness dimension or the tensile-strength dimension or the principal-curvature dimensions or . . . There is no single, common measure of *property* of a solid that all specific properties of solids have in common. Rand supposed in error that there were, for she supposed it always the case that there is some same, common measurable dimension supporting the conceptual common

> denominator for any super-ordinate concept. That supposition is here rejected, and measurement-omission analysis of super-ordinate concepts is here corrected in this respect. (283)

Here he seems to agree with me—that Rand erred in supposing there is always some same measurable dimension supporting the conceptual common denominator for any superordinate concept. I take it further—that it is not only superordinate concepts.

> But there is one form of challenge that is invalid, and I want to draw attention to this fallacy, which has required a long struggle for me to overcome. That is a fallacy I insinuated in Boydstun 1990, 33–34. It says that because preschoolers do not possess—not even tacitly—mathematical understanding sufficient to be forming their concepts using a principle of measurement-omission, their concepts do not bear analysis in terms of measurement-omission. That is the fallacy of confusing genesis with analysis. (294)

What exactly Boydstun means by this fallacy is not clear to me, but in my view it is descriptive of Rand's confounding. Being able to analyze a concept on a certain criterion later on does not imply that criterion was material, or was recognized when the concept was formed. Also, here Boydstun seems to go against the above excerpt from his page 283, at least when the child forms a superordinate concept.

6. Piaget also tested children's ability to judge whether quantities of liquids differed or were the same. The liquids were in different containers so shaped that one could not estimate their ratios by direct perception. There were extra empty containers that could be used to determine if the liquid in one container was less than, equal to, or greater than the liquid in another container. In the first case, no hint is given to the child about using one of the extra containers as a unit to measure. In the second case, the child is instructed in such use. Similar to length the children were not capable of measuring until about age 8. These experiments were about volume or "liquid amount," rather than length, but the results again show the large difference between perception of quantity and the *concept* of measurement (Gruber and Vonèche 1995, 330–34).

References

Aristotle. 1952. *Metaphysics*. Translated by W. D. Ross. Great Books of the Western World, Vol. 8. Chicago: Encyclopedia Britannica.

Boydstun, Stephen. 1990. Capturing concepts. *Objectivity* 1, no 1: 13–41.

___. 2004. Universals and measurement. *The Journal of Ayn Rand Studies* 5, no. 2 (Spring): 271–305.

Campbell, Norman. [1921] 1988. Measurement. In *The World of Mathematics*, Vol. 3, edited by J. Newman. Redmond, Washington: Tempus Books of Microsoft Press.

Campbell, Robert L. 2000. Implied epistemology, epistemology of the implicit. *The Journal of Ayn Rand Studies* 2, no. 1 (Fall): 211–19.

Euclid. 1952. *Elements*. Translated by Sir Thomas L. Heath. Great Books of the Western World, Vol. 11. Chicago: Encyclopedia Britannica.

Gelman, Rochel, and E. Williams. 1998. Enabling constraints for cognitive development and learning: Domain specificity and epigenesis. In *Handbook of Child Psychology*, 5th edition, vol. 2, edited by William Damon (chief), Deanna Kuhn and Robert S. Siegler. New York: John Wiley & Sons, 575–630.

Gruber, Howard, and Jacques Vonèche. 1995. *The Essential Piaget*. Northvale, New Jersey: Aronson

Inhelder, Bärbel, and Jean Piaget. [1959] 1964. *The Early Growth of Logic in the Child*. New York: Humanities Press.

Jetton, Merlin. 1990. The nature of numbers. *Objectivity* 1, no. 1: 1–12.

Kelley, David. 1984. A theory of abstraction. *Cognition and Brain Theory* 7: 329–57.

Locke, John. [1690] 1952. *An Essay Concerning Human Understanding*. Edited by Alexander Campbell Fraser. Great Books of the Western World, Vol. 35. Chicago: Encyclopedia Britannica.

Michell, Joel. 1999. *Measurement in Psychology*. Cambridge: Cambridge University Press.

Peikoff, Leonard. [1991] 1993. *Objectivism: The Philosophy of Ayn Rand*. New York: Meridian.

Piaget, Jean. [1941] 1965. *The Child's Conception of Number*. New York: W. W. Norton and Company.

Piaget, Jean, Bärbel Inhelder, and Alina Szeminska. [1948] 1960. *The Child's Conception of Geometry*. New York: W. W. Norton & Company.

Rand, Ayn. [1966] 1990. *Introduction to Objectivist Epistemology*. Expanded 2nd edition. Edited by Harry Binswanger and Leonard Peikoff. New York: Meridian.

___. [1997] 1999. *Journals of Ayn Rand*. Edited by David Harriman. New York: Penguin Putnam.

Register, Bryan. 2000. The universality and employment of concepts. *The Journal of Ayn Rand Studies* 1, no. 2 (Spring): 211–44.

Stevens, S. S. 1971. Issues in psychophysical measurement. *Psychological Review* 78, no. 5: 426–50.